THE WE-HERAEUS INTERNATIONAL WINTER SCHOOL ON

GRAVITY AND LIGHT

Integration

Exercise 1: Integrals and Volumes

Five basic questions (sparking discussion, you know) and one basic calculation.

Tick the correct statements, but not the incorrect ones!

- ○ Integration on a manifold requires a notion of orientability.
- ○ The integral over some chart domain U of a function f on the manifold is simply defined in the chart (U, x) as $\int_U f = \int_{x(U)} d^d\alpha f_{(x)}(\alpha)$.
- ○ Any volume form can be chosen to integrate over an oriented manifold.
- ○ The transformation law for integrals also holds for transition between charts.
- ○ The component functions of the volume form satisfy $\Omega_{i_1 i_2 \ldots i_d} = \Omega_{[i_1 i_2 \ldots i_d]}$.

Question: Calculate the volume of the round sphere S^2 of radius R, i.e.,

$$\mathrm{vol}(S^2) = \int_{S^2} 1.$$

Solution: