THE WE-HERAEUS INTERNATIONAL WINTER SCHOOL ON

GRAVITY AND LIGHT

Diagrams

Exercise 1: True or false

These basic questions are designed to spark discussion and as a self-test.

Which statements about a Schwarzschild black hole are correct?

- ○ In Schwarzschild coordinates, null geodesics starting at $r > 2m$ never reach $r < 2m$.
- ○ A Schwarzschild black hole is a non-rotating, but charged black hole.
- ○ In Eddington-Finkelstein coordinates, radial null geodesics fall into the black hole on straight lines at angle 45°.
- ○ Eddington-Finkelstein coordinates cover all possible radii $r \in (0, \infty)$.
- ○ Null geodesics in Schwarzschild coordinates starting from $r < 2m$ can never leave this region.

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Exercise 2: Null geodesics

Conformal transformations of the metric have no effect on the geodesic equation.

Question: Show that a curve γ is a null geodesic with respect to a metric g if, and only if, it is a null geodesic with respect to $\Omega^2 g$, where $\Omega^2 \in \mathcal{C}^\infty(M)$ is nowhere vanishing.

Solution:

Diagrams

Exercise 3: Penrose diagram of a radiation-filled universe

Understanding the causal structure of a homogeneous and isotropic universe.

Question: Find a differential equation for radial null geodesics in a spatially flat FRW universe filled with radiation, using the chart (t, r, ϑ, ϕ) introduced in the lectures. Explicitly write down the precise range of the chart variables.

Solution:

Question: Determine the t-coordinate of a geodesic in terms of its r-coordinate. Draw some of the null geodesics in the underlying chart.

Solution:

Question: Find a chart in which the geodesics are lines of constant slope ± 1. Determine the range of the coordinates.

Solution:

Question: Choose the so-called null coordinates u and v in which the null geodesics of positive slope are parallel to the u-axis and the ones of negative slope are parallel to the v-axis. Determine the range of the coordinates.

Solution:

Question: Compactify, i.e., rescale to finite ranges, each of the two null coordinates by an appropriate transformation. Determine the range of the coordinates.

Solution:

Question: By a final transformation, recover the notion of temporal and radial coordinates. Determine the range of those coordinates. Draw the Penrose-Carter diagram.

Solution: