The Theoretical Minimum

Classical Mechanics - Solutions

I01E03

Last version: tales.mbivert.com/on-the-theoretical-minimum-solutions/ or github.com/mbivert/ttm

M. Bivert

May 10, 2023

Exercise 1. *Show that the magnitude of a vector satisfies* $|\vec{A}|^2 = \vec{A} \cdot \vec{A}$.

Remark 1. *We'll again use a* ***bold*** *font to denote vectors instead of arrows and use a slightly different symbol for the magnitude; the change can be summed up by stating:* $\|\boldsymbol{u}\| = |\vec{u}|\,(= u)$.

Let's recall that the magnitude of a vector was *defined* as:

$$\|\boldsymbol{u}\| = \sqrt{u_x^2 + u_y^2 + u_z^2}$$

And the dot product between two vectors as:

$$\boldsymbol{u} \cdot \boldsymbol{v} = u_x v_x + u_y v_y + u_z v_z$$

From there, we quickly reach the expected result:

$$\begin{aligned} \boldsymbol{u} \cdot \boldsymbol{u} &= u_x u_x + u_y u_y + u_z u_z \\ &= u_x^2 + u_y^2 + u_z^2 \\ &= \left(\sqrt{u_x^2 + u_y^2 + u_z^2}\right)^2 \\ &= \|\boldsymbol{u}\|^2 \quad \square \end{aligned}$$