# The Theoretical Minimum

## Classical Mechanics - Solutions

### I01E04

Last version: tales.mbivert.com/on-the-theoretical-minimum-solutions/ or github.com/mbivert/ttm

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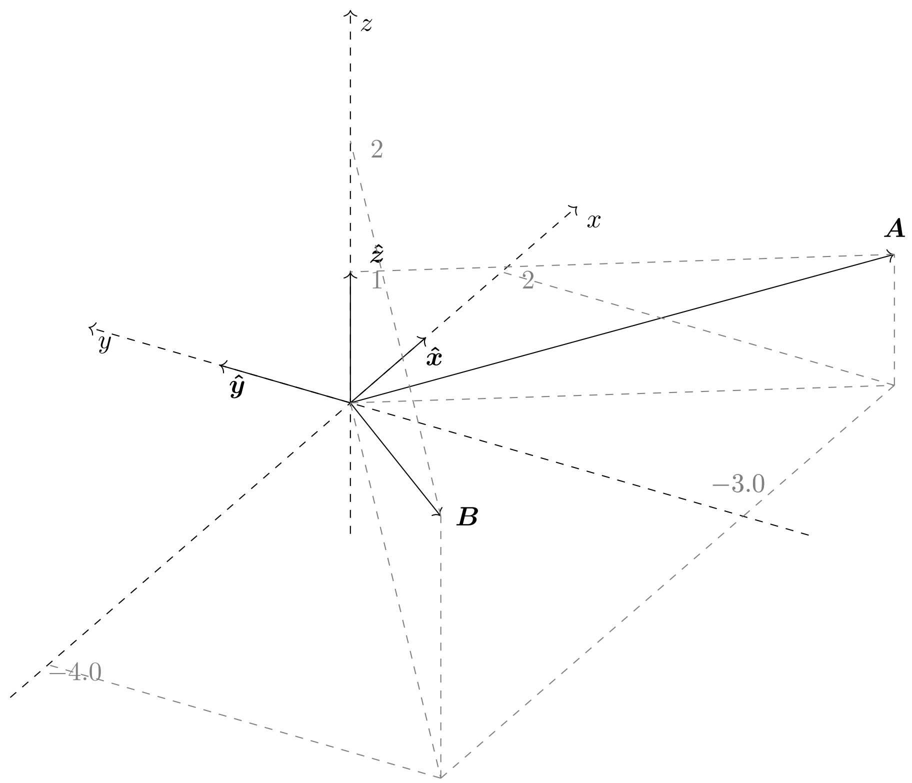
**Exercise 1.** *Let* $(A_x = 2, A_y = -3, A_z = 1)$ *and* $(B_x = -4, B_y = -3, B_z = 2)$*. Compute the magnitude of* $\vec{A}$ *and* $\vec{B}$*, their dot product, and the angle between them.*

This is an immediate application of the formulas. Let's start by plotting those vectors:




Using slightly different notations, let us then recall first how the magnitude of a vector $\boldsymbol{u}$ is defined:

$$\|\boldsymbol{u}\| = \sqrt{u_x^2 + u_y^2 + u_z^2} = u \tag{1}$$

There are two formulas for the dot product: one involving the magnitudes and the angle between the vectors, and the other one involving the components:

$$\begin{aligned} \boldsymbol{u} \cdot \boldsymbol{v} &= u_x v_x + u_y v_y + u_z v_z \\ &= \|\boldsymbol{u}\| \|\boldsymbol{v}\| \cos \theta_{uv} \end{aligned} \tag{2}$$

By applying (1), we have:

$$
\begin{aligned}
A &= \sqrt{2^2 + (-3)^2 + 1^2} & \quad B &= \sqrt{(-4)^2 + (-3)^2 + 2^2} \\
&= \sqrt{4 + 9 + 1} & &= \sqrt{16 + 9 + 4} \\
&= \boxed{\sqrt{14}} & &= \boxed{\sqrt{29}}
\end{aligned}
$$

We can also compute the dot product from the vectors' components, using the first form of (2):

$$
\begin{aligned}
\boldsymbol{A} \cdot \boldsymbol{B} &= 2(-4) + (-3)(-3) + 1 \times 2 \\
&= \boxed{3}
\end{aligned}
$$

From the second form of (2), we can deduce a formula for the angle between $\boldsymbol{A}$ and $\boldsymbol{B}$, $\theta_{AB}$:

$$
\begin{aligned}
\cos\theta_{AB} &= \frac{\boldsymbol{A} \cdot \boldsymbol{B}}{\|\boldsymbol{A}\|\|\boldsymbol{B}\|} \\
\Leftrightarrow \quad \theta_{AB} &= \cos^{-1}\left(\frac{\boldsymbol{A} \cdot \boldsymbol{B}}{\|\boldsymbol{A}\|\|\boldsymbol{B}\|}\right) \\
\Leftrightarrow \quad &= \cos^{-1}\left(\frac{3}{\sqrt{14 \times 29}}\right) \\
\Leftrightarrow \quad &= \boxed{81.43753893^\circ}
\end{aligned}
$$

**Remark 1.** *Looking at our plots, the angle feels to be something a little less than* $90^\circ$*, which is coherent with what we've found.*