The Theoretical Minimum

Classical Mechanics - Solutions

I01E05

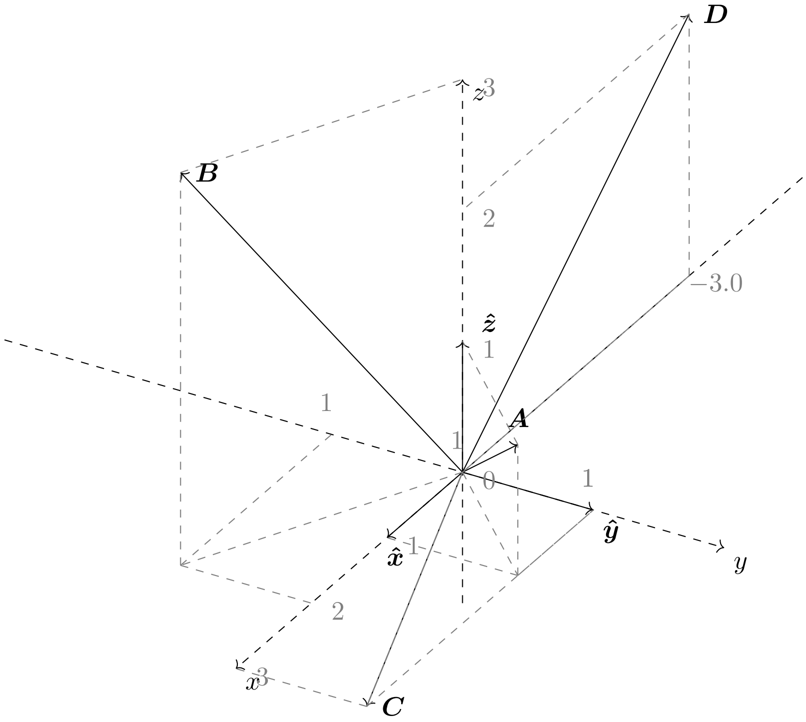
Last version: tales.mbivert.com/on-the-theoretical-minimum-solutions/ or github.com/mbivert/ttm

M. Bivert

May 10, 2023

Exercise 1. *Determine which pair of vectors are orthogonal.* $(1, 1, 1), (2, -1, 3), (3, 1, 0), (-3, 0, 2)$.

This is again an immediate application of dot product formula. We'll respectively name the vectors $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$; let's start by plotting them:



Right before this exercise, the authors wrote:

> An important property of the dot product is that it is zero if the vectors are *orthogonal.*

Let us recall the "components-based" dot product formula we'll be using:

$$\boldsymbol{u} \cdot \boldsymbol{v} \quad = \quad u_x v_x + u_y v_y + u_z v_z$$

Then, it's just a matter of crunching numbers:

$$\begin{aligned}
\boldsymbol{A}\cdot\boldsymbol{B} &= A_xB_x + A_yB_y + A_zB_z \\
&= (1\times 2) + (1\times(-1)) + (1\times 3) \\
&= \boxed{4}
\end{aligned}
\qquad
\begin{aligned}
\boldsymbol{A}\cdot\boldsymbol{C} &= A_xC_x + A_yC_y + A_zC_z \\
&= (1\times 3) + (1\times 1) + (1\times 0) \\
&= \boxed{4}
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{A}\cdot\boldsymbol{D} &= A_xD_x + A_yD_y + A_zD_z \\
&= (1\times(-3)) + (1\times 0) + (1\times 2) \\
&= \boxed{-1}
\end{aligned}
\qquad
\begin{aligned}
\boldsymbol{B}\cdot\boldsymbol{C} &= B_xC_x + B_yC_y + B_zC_z \\
&= (2\times 3) + (-1\times 1) + (3\times 0) \\
&= \boxed{5}
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{B}\cdot\boldsymbol{D} &= B_xD_x + B_yD_y + B_zD_z \\
&= (2\times(-3)) + (-1\times 0) + (3\times 2) \\
&= \boxed{0}
\end{aligned}
\qquad
\begin{aligned}
\boldsymbol{D}\cdot\boldsymbol{C} &= D_xC_x + D_yC_y + D_zC_z \\
&= (-3\times 3) + (0\times 1) + (2\times 0) \\
&= \boxed{-9}
\end{aligned}$$

Hence, the only two orthogonal vectors are $\boldsymbol{B}$ and $\boldsymbol{D}$, or $(2,-1,3)$ and $(-3,0,2)$.