The Theoretical Minimum

Classical Mechanics - Solutions

I02E02

Last version: tales.mbivert.com/on-the-theoretical-minimum-solutions/ or github.com/mbivert/ttm

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Exercise 1. *Use the fundamental theorem of calculus to evaluate each integral from Exercise 1 with limits of integration being $t = 0$ to $t = T$.*

We're going to build on the indefinite integrals we've just computed in I02E01, and simply evaluate the difference of the primitives between $t = T$ and $t = 0$.

Remark 1. *There are two common notations to evaluate a primitive between two values; I'll use the second one, out of habit. Let's recall the fundamental theorem of calculus along the way:*

$$\boxed{F(t)\big|_a^b = [F(t)]_a^b \triangleq \int_a^b F'(t)\,dt = F(b) - F(a)}$$

$f(t) = t^4$
The primitive was:

$$\frac{1}{5}t^5 + c, \quad c \in \mathbb{R}$$

Evaluated as expected gives:

$$\left[\frac{1}{5}t^5 + c\right]_0^T = \frac{1}{5}T^5 + c - (\frac{1}{5}0^5 + c) = \boxed{\frac{1}{5}T^5}$$

Remark 2. *Note how the constant of integration gets canceled. This will happen systematically here.*

$f(t) = \cos t$
The primitive was:

$$\sin t + c, \quad c \in \mathbb{R}$$

Evaluated as expected gives:

$$[\sin t + c]_0^T = \sin T - \underbrace{\sin 0}_{=0} = \boxed{\sin T}$$

$f(t) = t^2 - 2$
The primitive was:

$$\frac{1}{3}t^3 - 2t + c, \quad c \in \mathbb{R}$$

Evaluated as expected gives:

$$\left[\frac{1}{3}t^3 - 2t + c\right]_0^T = \boxed{\frac{1}{3}T^3 - 2T}$$