The Theoretical Minimum

Classical Mechanics - Solutions

I02E04

Last version: tales.mbivert.com/on-the-theoretical-minimum-solutions/ or github.com/mbivert/ttm

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Exercise 1. *Finish evaluating*

$$\int_0^{\frac{\pi}{2}} x \cos x \, dx$$

We will re-compute the integral from the beginning.

Let's start by recalling the formula for the integration by parts, given two real valued functions φ and ψ continuously differentiable on $[a, b] \subset \mathbb{R}$:

$$\boxed{\int_a^b \varphi'(x)\psi(x) \, dx = [\varphi(x)\psi(x)]_a^b - \int_a^b \varphi(x)\psi'(x) \, dx}$$

Now, in our integral, let's identify:

$$\begin{aligned} \varphi'(x) &= x \\ \psi(x) &= \cos x \end{aligned}$$

From which we have immediately:

$$\begin{aligned} \varphi(x) &= \frac{1}{2}x^2 \\ \psi'(x) &= \sin x \end{aligned}$$

Our integral then becomes:

$$\int_0^{\pi/2} x \cos x \, dx = \left[\frac{1}{2}x^2 \cos x\right]_0^{\pi/2} - \int_0^{\pi/2} \frac{1}{2}x^2 \sin x \, dx$$

Well, it didn't really helped us, didn't it? But this gives us a clue on what perhaps we should try: while the sine/cosine will essentially oscillate between each other when integrating/differentiating, the x will square when being integrated, but will vanish when being differentiated. So instead, let's identify things this way:

$$\begin{aligned} \varphi'(x) &= \cos x; & \varphi(x) &= \sin x \\ \psi(x) &= x; & \psi'(x) &= 1 \end{aligned}$$

And now the integral becomes:

$$\begin{aligned} \int_0^{\pi/2} x \cos x \, dx &= [x \sin x]_0^{\pi/2} - \int_0^{\pi/2} \sin x \, dx \\ &= \frac{\pi}{2} + [\cos x]_0^{\pi/2} \\ &= \boxed{\frac{\pi}{2} - 1} \end{aligned}$$