The Theoretical Minimum

Classical Mechanics - Solutions

L01E03

Last version: tales.mbivert.com/on-the-theoretical-minimum-solutions/ or github.com/mbivert/ttm

M. Bivert

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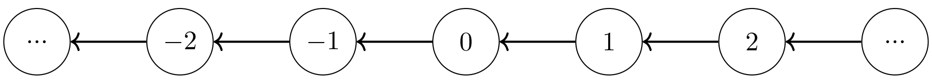
Exercise 1. *Determine which of the dynamical laws shown in Eq.s* (2) *through* (5) *are allowable.*

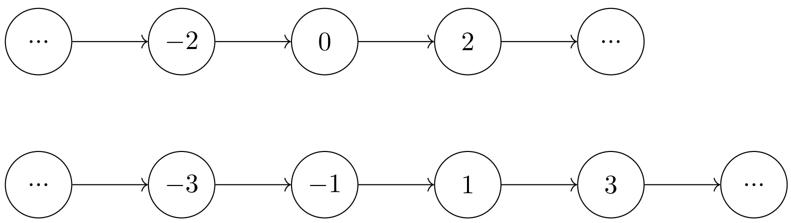
Let's recall that laws are said to be allowable if they are both deterministic (i.e. the future behavior of the system is completely determined by the initial state) and reversible (i.e. the law is still deterministic even by reversing the direction of all the arrows).

Now, the dynamical laws referred to be the exercise are:

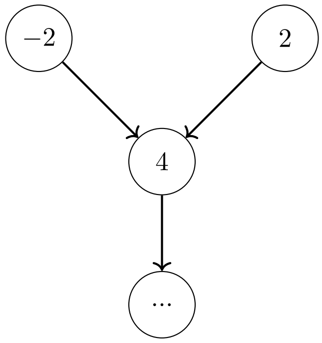
$$\begin{aligned} N(n+1) &= N(n)-1 \qquad (2) \\ N(n+1) &= N(n)+2 \qquad (3) \\ N(n+1) &= N(n)^2 \qquad (4) \\ N(n+1) &= -1^{N(n)}N(n) \qquad (5) \end{aligned}$$



(2) is simply (1) (described by $N(n+1) = N(n)+1$) with reversed arrows; and (1) has already be established as being allowed. Hence, $\boxed{\text{(2) is allowed}}$.

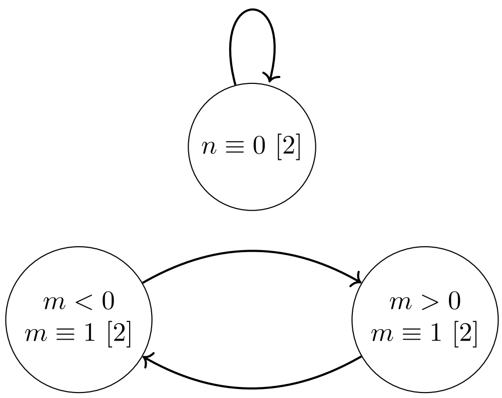


(3) is very similar to (1) or (2), only, it has two infinite cycles instead of one. Both are deterministic, and reversing the arrows lead to deterministic cycles. Hence, $\boxed{\text{(3) is allowed}}$.



We didn't plot (4) exhaustively, just enough to illustrate its irreversibility: if we start at either -2 or 2, we end up on 4. By reversing the arrows, we see that we can't decide where we came from. Furthermore, states such as -2 cannot be reached, because there's no number n such as $-2 = n^2$). Actually, even 2 cannot be reached, because there's no *integer* n such as $2 = n^2$.

Hence, (4) is **not** allowed.



By sketching a few cycles for (5), we see a pattern emerging, summed up in the diagram above:

- if we start on an even number, positive or negative, we'll loop on this number indefinitely;
- if we start with an odd number, positive or negative, we'll loop between the positive and negative version of that number;

Perhaps a mathematical subtlety would be in considering -1 raised to a negative power. But,

$$(\forall n \in \mathbb{N}^*), \quad -1^{-n} = (-1/1)^{-n} \equiv (1/-1)^n = (-1)^n$$

Hence, (5) is allowed.