The Theoretical Minimum

Classical Mechanics - Solutions

L02E02

Last version: tales.mbivert.com/on-the-theoretical-minimum-solutions/ or github.com/mbivert/ttm

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Exercise 1. *The derivative of a derivative is called the second derivative and is written* $\frac{d^2f(t)}{dt^2}$. *Take the second derivative of each of the functions listed above.*

Remark 1. *As for the first derivative, there are two common notations for second derivatives:*

$$\frac{d^2}{dt^2}\varphi = \varphi'' = \ddot{\varphi}$$

Again, the dot notation is used in physics to denote time differentiation, while the prime notation is often used in mathematics, in a more abstract context.

Let's start by recalling the functions:

$$\begin{aligned} f(t) &= t^4 + 3t^3 - 12t^2 + t - 6 \\ g(x) &= \sin x - \cos x \\ \theta(\alpha) &= e^\alpha + \alpha \ln \alpha \\ x(t) &= \sin^2 t - \cos t \end{aligned}$$

And their derivatives, computed in the previous exercise:

$$\begin{aligned} f'(t) &= 4t^3 + 9t^2 - 24t + 1 \\ g'(x) &= \cos(x) + \sin(x) \\ \theta'(\alpha) &= e^\alpha + \ln(\alpha) + 1 \\ x'(t) &= (1 + 2\cos t)\sin t \end{aligned}$$

While in the previous exercise (L02E01) the derivation were rather slow and detailed, because the process is essentially the same, we're going to go (much) faster here, the most difficult part being in the application of the product rule for x''.

$$\begin{aligned} f''(t) &= \boxed{12t^2 + 18t - 24} \\ g''(x) &= \boxed{\cos(x) - \sin(x) = -g(x)} \\ \theta''(\alpha) &= \boxed{e^\alpha + \frac{1}{\alpha}} \\ x''(t) &= (1 + 2\cos t)\cos t - 2\sin t \sin t \\ &= \boxed{\cos t + 2(\cos^2 t - \sin^2 t)} \end{aligned}$$

Remark 2. x'' *could be slightly improved by using the trigonometric identity* $\cos^2 x - \sin^2 x = \cos 2x$, *which hasn't been introduced in the book.*