The Theoretical Minimum

Classical Mechanics - Solutions

L02E06

Last version: tales.mbivert.com/on-the-theoretical-minimum-solutions/ or github.com/mbivert/ttm

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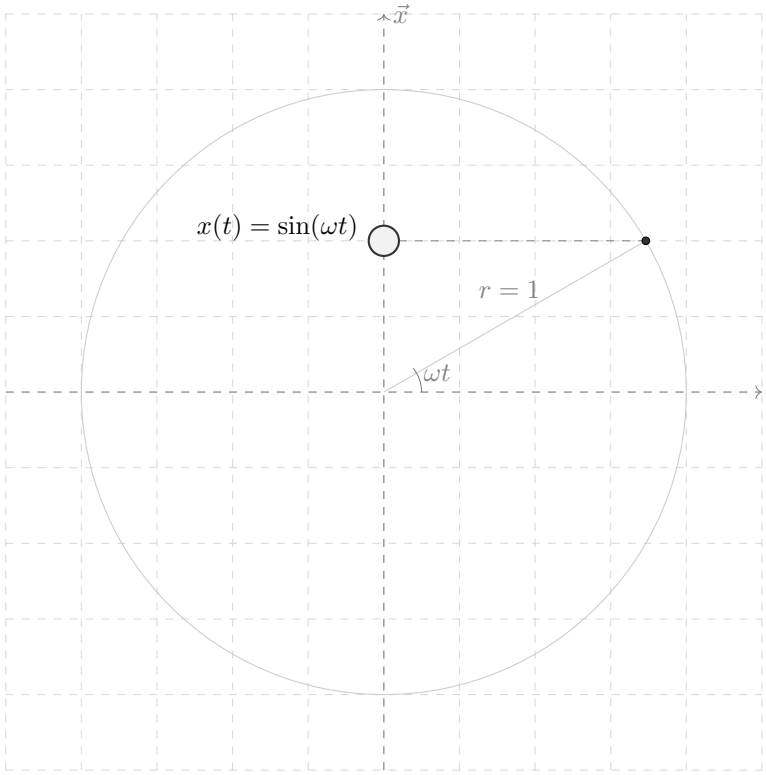
Exercise 1. *How long does it take for the oscillating particle to go through one full cycle of motion?*

We're in the case of a particle oscillating in one dimension. Its motion, known as the *simple harmonic motion*, is described by:

$$x(t) = \sin(\omega t)$$

Essentially, $x(t)$ will correspond to the vertical component of a point moving on the unit circle, located by an angle ωt.



To fix things, consider the case of a particle starting at an extreme position, say $x = 1$ (at the top of the north hemisphere of the unit circle). It will need to go down to $x = -1$, and then back up to $x = 1$. In the mean time, the corresponding point on the unit circle would have walked a full circle, or 2π radians.

So we're looking for the time T that it will take for us to move by an angle 2π, knowing that we move at a speed of ω radians per unit of time (i.e. $\omega_{t=0} = 0$, $\omega_{t=1} = \omega$, $\omega_{t=2} = 2\omega$, ...):

$$\omega T = 2\pi \Leftrightarrow \boxed{T = \frac{2\pi}{\omega}}$$

Remark 1. *T is commonly called the period of motion.*