# The Theoretical Minimum

## Classical Mechanics - Solutions

### L03E01

Last version: tales.mbivert.com/on-the-theoretical-minimum-solutions/ or github.com/mbivert/ttm

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**Exercise 1.** *Given a force that varies with time according to $F = 2t^2$, and with the initial condition at time zero, $x(0) = \pi$, use Aristotle's law to find $x(t)$ at all times.*

Let us recall that Aristotle's law of motion is defined, for a one-dimensional particle (otherwise, $F(t)$ and $x(t)$ would be vector-values functions $\boldsymbol{F}(t)$ and $\boldsymbol{x}(t)$) earlier in the book as:

$$\frac{d}{dt}x(t) = \frac{F(t)}{m}$$

And that by integrating both sides, thanks to the fundamental theorem of calculus[1], assuming the mass is constant over time, we obtain:

$$x(t) = \frac{1}{m}\int F(t)\,dt$$

Which is our case, for $F(t) = 2t^2$, develops in:

$$\begin{aligned} x(t) &= \frac{1}{m}\int 2t^2\,dt \\ &= \frac{2}{3m}t^3 + c,\ c \in \mathbb{R} \end{aligned}$$

The initial condition $x(0) = \pi$ implies that $c = \pi$, hence the position at all time would be:

$$\boxed{x(t) = \frac{2}{3m}t^3 + \pi} \quad \square$$

[1] https://en.wikipedia.org/wiki/Fundamental_theorem_of_calculus