The Theoretical Minimum

Classical Mechanics - Solutions

L06E02

Last version: tales.mbivert.com/on-the-theoretical-minimum-solutions/ or github.com/mbivert/ttm

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Exercise 1. *Show that Eq.* (6) *is just another form of Newton's equation of motion* $F_i = m_i\ddot{x}_i$.

Where Eq. (6) are the following set of equation, defined for all $i \in [\![1, n]\!]$:

$$\frac{d}{dt}\left(\frac{\partial}{\partial \dot{x}_i}L\right) = \frac{\partial}{\partial x_i}L \tag{1}$$

Remark 1. *This exercise is simply a generalization of the previous exercise (L06E01) to a configuration space of size* $n \in \mathbb{N}$.

Then again, let us recall the Lagrangian defined slightly earlier in the related section of the book:

$$L = \sum_{i=1}^{n}\left(\frac{1}{2}m_i\dot{x_i}^2\right) - V(\{x\}) \tag{2}$$

Hence, $(\forall i \in [\![1, n]\!])$:

$$\begin{aligned}\frac{\partial}{\partial \dot{x}_i}L &= \frac{\partial}{\partial \dot{x}_i}\sum_{j=1}^{n}\frac{1}{2}m_j\dot{x_j}^2 \\ &= \sum_{j=1}^{n}m_j\dot{x_j}\delta_{ij} \\ &= m_i\dot{x_i}\end{aligned} \qquad \frac{\partial}{\partial x_i}L = -\frac{\partial}{\partial x_i}V(\{x\}) \tag{3}$$

Again, we need the *potential energy principle*, stated as Eq. (5) of the previous chapter *Lecture 5: Energy*, for abstract configuration space $\{x\} = \{x_i\}$, as:

$$F_i(\{x\}) = -\frac{\partial}{\partial x_i}V(\{x\}) \tag{4}$$

From which we can conclude, by injecting (4) in the second half of (3), and connecting each side with Euler-Lagrange's equations (1), $(\forall i \in [\![1, n]\!])$:

$$\begin{aligned}&\frac{d}{dt}\left(\frac{\partial}{\partial \dot{x}_i}L\right) = \frac{\partial}{\partial x_i}L \\ \Leftrightarrow &\frac{d}{dt}m_i\dot{x}_i = F_i(\{x\}) \\ \Leftrightarrow &\boxed{F_i = m_i\ddot{x}_i} \quad \square\end{aligned}$$