# The Theoretical Minimum

## Classical Mechanics - Solutions

### L11E02

Last version: tales.mbivert.com/on-the-theoretical-minimum-solutions/ or github.com/mbivert/ttm

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**Exercise 1.** *Prove Eq.* (4).

Where Eq. (4) is the following, for $V$ a scalar field:

$$\vec{\nabla} \times \left(\vec{\nabla} V(x)\right) = 0$$

If think we can agree that $V(x)$ is actually a $V(x, y, z)$.

And $\vec{\nabla}$ is the differentiation vector operator:

$$\vec{\nabla} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}$$

By this definition,

$$\vec{\nabla} V(x, y, z) = \begin{pmatrix} \frac{\partial V}{\partial x} \\ \frac{\partial V}{\partial y} \\ \frac{\partial V}{\partial z} \end{pmatrix}$$

We also have previously established that for a field $F = (F_x, F_y, F_z)$:

$$\vec{\nabla} \times F = \begin{pmatrix} \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \\ \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \\ \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \end{pmatrix}$$

And so,

$$\vec{\nabla} \times (\vec{\nabla} V(x, y, z)) = \begin{pmatrix} \frac{\partial}{\partial y}\frac{\partial V}{\partial z} - \frac{\partial}{\partial z}\frac{\partial V}{\partial y} \\ \frac{\partial}{\partial z}\frac{\partial V}{\partial x} - \frac{\partial}{\partial x}\frac{\partial V}{\partial z} \\ \frac{\partial}{\partial x}\frac{\partial V}{\partial y} - \frac{\partial}{\partial y}\frac{\partial V}{\partial x} \end{pmatrix} = \vec{0}$$

Where we can conclude because of Schwarz/Clairaut's theorem [1]. This means we consider $V$ to have continuous second partial derivatives on its domain (or, at least in a neighborhood of a point $x$ of its domain), which is often a reasonable assumption in Physics.

[1] `https://en.wikipedia.org/wiki/Symmetry_of_second_derivatives`