The Theoretical Minimum

Quantum Mechanics - Solutions

L01E01

Last version: tales.mbivert.com/on-the-theoretical-minimum-solutions/ or github.com/mbivert/ttm

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Exercise 1. *a) Using the axioms for inner products, prove*

$$\Big(\langle A| + \langle B|\Big)|C\rangle = \langle A|C\rangle + \langle B|C\rangle$$

b) Prove $\langle A|A\rangle$ is a real number.

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a) Let us recall the two axioms in question:

Axiom 1.

$$\langle C|\Big(|A\rangle + |B\rangle\Big) = \langle C|A\rangle + \langle C|B\rangle$$

Axiom 2.

$$\langle B|A\rangle = \langle A|B\rangle^*$$

Where z^* is the complex conjugate of $z \in \mathbb{C}$

Let us recall also that if

- $\langle A|$ is the bra of $|A\rangle$
- $\langle B|$ is the bra of $|B\rangle$

Then $\langle A| + \langle B|$ is the bra of $|A\rangle + |B\rangle$.

Let us also observe that for $(a, b) = (x_a + iy_a, x_b + iy_b) \in \mathbb{C}^2$:

$$\begin{aligned}(a+b)^* &= (x_a + iy_a + x_b + iy_b)^* \\ &= x_a - iy_a + x_b - iy_b \\ &= a^* + b^*\end{aligned}$$

We thus have:

$$\begin{aligned}\Big(\langle A| + \langle B|\Big)|C\rangle &= \Big(\langle C|\Big(|A\rangle + |B\rangle\Big)\Big)^* \\ &= \Big(\langle C|A\rangle + \langle C|B\rangle\Big)^* \\ &= \langle C|A\rangle^* + \langle C|B\rangle^* \\ &= \langle A|C\rangle + \langle B|C\rangle \quad \square\end{aligned}$$

b) Mainly from the second axiom:

$$\begin{aligned}
x + iy &= \langle A|A\rangle \\
&= \langle A|A\rangle^* \\
&= x - iy \\
&\Rightarrow 2iy = 0 \\
&\Rightarrow y = 0 \\
&\Rightarrow \langle A|A\rangle = x \in \mathbb{R} \quad \square
\end{aligned}$$