The Theoretical Minimum

Quantum Mechanics - Solutions

L04E01

Last version: tales.mbivert.com/on-the-theoretical-minimum-solutions/ or github.com/mbivert/ttm

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Exercise 1. *Prove that if U is unitary, and if $|A\rangle$ and $|B\rangle$ are any two state-vectors, then the inner product of $U|A\rangle$ and $U|B\rangle$ is the same as the inner product of $|A\rangle$ and $|B\rangle$. One could call this the conservation of overlaps. It expresses the fact that the logical relation between states is preserved with time.*

The inner-product has been defined as the product of a bra and a ket. So the inner-product of $U|A\rangle$ and $U|B\rangle$ is the product of e.g. the bra associated to $U|A\rangle$ and $U|B\rangle$. But in section 3.1.5 of the book, we've established that:

$$|C\rangle = M|D\rangle \Leftrightarrow \langle C| = \langle D|M^\dagger$$

Hence the inner-product we're looking for is:

$$\langle A|\underbrace{U^\dagger U}_{I}|B\rangle = \langle A|B\rangle \quad \square$$

Remark 1. *The terminology is a bit confusing: we're talking about the inner-product of two kets, while we've defined the inner-product to be an operation between a bra and a ket. Overall, the bra-ket notation makes things a little more complicated than just having to deal with an inner-product space.*