The Theoretical Minimum

Quantum Mechanics - Solutions

L04E02

Last version: tales.mbivert.com/on-the-theoretical-minimum-solutions/ or github.com/mbivert/ttm

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May 10, 2023

Exercise 1. *Prove that if M and L are both Hermitian, $i[M,L]$ is also Hermitian. Note that the i is important. The commutator is, by itself, not Hermitian.*

$$
\begin{aligned}
(i[M,L])^\dagger &= (i(ML-LM))^\dagger \\
&= (iML - iLM)^\dagger \\
&= \left((iML-iLM)^T\right)^* && (\dagger\text{'s definition}) \\
&= \left((iML)^T - (iLM)^T\right)^* && \left((A+B)^T = A^T + B^T\right) \\
&= \left(iL^TM^T - iM^TL^T\right)^* && \left((AB)^T = B^TA^T\right) \\
&= \left(-i(L^TM^T)^* + i(M^TL^T)^*\right) && ((zt)^* = z^*t^*; (z+t)* = z^* + t^*)) \\
&= i\left(M^\dagger L^\dagger - L^\dagger M^\dagger\right) && (\dagger\text{'s definition}) \\
&= i\left(ML - LM\right) && \left(L = L^\dagger; M = M^\dagger\right) \\
&= i[M,L] && \square
\end{aligned}
$$