The Theoretical Minimum
Quantum Mechanics - Solutions
L06E06

Last version: tales.mbivert.com/on-the-theoretical-minimum-solutions/ or github.com/mbivert/ttm

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Exercise 1. *Assume Charlie has prepared the two spins in the singlet state. This time, Bob measures τ_y and Alice measure σ_x. What is the expectation value of $\sigma_x\tau_y$?*

What does this say about the correlation between the two measurements?

Let's recall the Pauli matrices involved:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \qquad \tau_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

And the singlet state:

$$|\text{sing}\rangle = \frac{1}{\sqrt{2}}\left(|ud\rangle - |du\rangle\right)$$

Recall:

$$\sigma_x|u\} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} =: |d\}; \qquad \sigma_x|d\} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} =: |u\}$$

$$\tau_y|u\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ i \end{pmatrix} =: i|d\rangle; \qquad \tau_y|d\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -i \\ 0 \end{pmatrix} =: -i|u\rangle$$

The expectation value of $\sigma_x\tau_y$ is then:

$$\begin{aligned}
\langle\sigma_x\tau_y\rangle := & \langle\text{sing}|\sigma_x\tau_y|\text{sing}\rangle \\
= & \langle\text{sing}|\sigma_x\tau_y\frac{1}{\sqrt{2}}\left(|ud\rangle - |du\rangle\right) \\
= & \frac{1}{\sqrt{2}}\langle\text{sing}|\left(\underbrace{(\sigma_x|u\})}_{|d\}}\otimes\underbrace{(\tau_y|d\rangle)}_{-i|u\rangle} - \underbrace{(\sigma_x|d\})}_{|u\}}\otimes\underbrace{(\tau_y|u\rangle)}_{i|d\rangle}\right) \\
= & \frac{-i}{\sqrt{2}}\langle\text{sing}|\left(|du\rangle + |ud\rangle\right) \\
= & \frac{-i}{2}(\langle ud| - \langle du|)(|du\rangle + |ud\rangle) \\
= & \frac{-i}{2}\left(\underbrace{\langle ud|du\rangle}_{0} + \underbrace{\langle ud|ud\rangle}_{1} - \underbrace{\langle du|du\rangle}_{1} - \underbrace{\langle du|ud\rangle}_{0}\right) \\
= & \boxed{0}
\end{aligned}$$

Remember for the last step that $|du\rangle$ and $|ud\rangle$ are orthonormal basis vectors.

On to the correlation between the two measurements: recall from section 6.2 that the statistical correlation between an observable σ_A in Alice's space and an observable σ_B in Bob's space was defined as the quantity:

$$\langle\sigma_A\sigma_B\rangle - \langle\sigma_A\rangle\langle\sigma_B\rangle$$

Hence in our case, the correlation between the two measurements is (the authors previously computed $\langle \sigma_x \rangle = 0$ and $\langle \sigma_y \rangle = 0$: the computation of $\langle \tau_y \rangle$ would be identical as for the latter)

$$\langle \sigma_x \tau_y \rangle - \langle \sigma_x \rangle \langle \tau_y \rangle = 0$$

Hence, we can conclude that the two measurements aren't correlated at all.