The Theoretical Minimum

Quantum Mechanics - Solutions

L07E06

Last version: tales.mbivert.com/on-the-theoretical-minimum-solutions/ or github.com/mbivert/ttm

M. Bivert

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Exercise 1. *Use Eq. 7.22 to show that if ρ is a density matrix, then*

$$Tr(\rho) = 1.$$

Eq. 7.22 is the following:

$$P(a) = \rho_{aa}$$

Where $P(a)$ is the probability for an observable L tied to Alice's state space, extended to act on a composite state-space made from Alice's and Bob's, to be measured with the eigenvalue a. On the other hand, ρ_{aa} corresponds to the diagonal elements of Alice's density matrix, expressed in Alice state space.

Well, there will be one $P(a)$ for each eigenvalue, and thus by Eq. 7.22, there is a systematic correspondence with the diagonal elements of the density matrix. But the trace of an operator is defined as the sum of the diagonal elements of a matrix representation of this operator, and it so happens that this value is unique up to a change of basis (meaning, the trace of an operator is the same for all matrix representation of this operator).

Hence because the eigenvalues a represent all the potential measurement values, we know that $\sum_a P(a) = 1$, which by our previous reasoning implies indeed that $\boxed{\text{Tr}(\rho) := \sum_a \rho_{aa} = \sum_a P(a) = 1}$ $\square$