The Theoretical Minimum

Quantum Mechanics - Solutions

L07E11

Last version: tales.mbivert.com/on-the-theoretical-minimum-solutions/ or github.com/mbivert/ttm

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Exercise 1. *Calculate Alice's density matrix for σ_z for the "near-singlet" state.*

The "near-singlet" state is characterized by the following state-vector:

$$|\Psi\rangle = \sqrt{0.6}|ud\rangle - \sqrt{0.4}|du\rangle$$

Alice's density matrix is defined by its components in Eq. 7.20[1]:

$$\rho_{a'a} = \sum_b \psi^*(a,b)\psi(a',b)$$

Where $\psi(a,b)$ is the wave-function of the composite system, that we can extract from $|\Psi\rangle$:

$$|\Psi\rangle = \psi(u,u)|uu\rangle + \psi(u,d)|ud\rangle + \psi(d,u)|du\rangle + \psi(d,d)|dd\rangle \Rightarrow \begin{cases} \psi(u,u) = \psi(d,d) & = 0 \\ \psi(u,d) & = \sqrt{0.6} \\ \psi(d,u) & = -\sqrt{0.4} \end{cases}$$

Hence:

$$\rho = \begin{pmatrix} \rho_{uu} & \rho_{ud} \\ \rho_{du} & \rho_{dd} \end{pmatrix} = \begin{pmatrix} \psi^*(u,u)\psi(u,u) + \psi^*(u,d)\psi(u,d) & \psi^*(d,u)\psi(u,u) + \psi^*(d,d)\psi(u,d) \\ \psi^*(u,u)\psi(d,u) + \psi^*(u,d)\psi(d,d) & \psi^*(d,u)\psi(d,u) + \psi^*(d,d)\psi(d,d) \end{pmatrix} = \boxed{\begin{pmatrix} 0.6 & 0 \\ 0 & 0.4 \end{pmatrix}} \quad \square$$

Remark 1. *I'm not sure what the authors expect regarding σ_z; we're asked to verify all numerical values in the next exercise, which likely should cover pretty much every intepretation (we'll even have to compute Alice's density matrix again, so as to check $\rho^2/Tr(\rho^2)$).*

[1] p205, section 7.5 *Entanglement for Two Spins*