The Theoretical Minimum

Quantum Mechanics - Solutions

L09E01

Last version: tales.mbivert.com/on-the-theoretical-minimum-solutions/ or github.com/mbivert/ttm

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Exercise 1. *Derive Eq.* 9.7 *by plugging Eq.* 9.6 *into Eq.* 9.5.

Let's recall in order, Eq. 9.7, Eq. 9.6 and Eq. 9.5:

$$E = p^2/2m; \qquad \psi(x) = \exp\left(ipx/\hbar\right); \qquad -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} = E\psi(x)$$

In that last equation, the RHS could be rewritten as $\boldsymbol{H}|\Psi\rangle$, where $\boldsymbol{H}$ is the "quantized" classical Hamiltonian corresponding to a free particle, that is, a particle not affected by a potential energy: the Hamiltonian is then built solely from the "quantized" kinetic energy.

Eq. 9.6 (the middle one) is a solution proposal to the ODE yielded by Eq. 9.5 (the last one). Let's see how it goes:

$$\begin{aligned}
& -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} = && E\psi(x) \\
\Leftrightarrow \qquad & -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\exp\left(ipx/\hbar\right) = && E\psi(x) \\
\Leftrightarrow \quad & -\frac{\hbar^2}{2m}\left(\frac{ip}{\hbar}\right)^2 \underbrace{\exp\left(ipx/\hbar\right)}_{=:\psi(x)} = && E\psi(x) \\
\Leftrightarrow \qquad & \frac{p^2}{2m}\psi(x) = && E\psi(x)
\end{aligned}$$

And so indeed, at least as long as $\psi(x) \neq 0$:

$$\boxed{E = p^2/2m} \quad \square$$