The Theoretical Minimum

Quantum Mechanics - Solutions

L09E03

Last version: tales.mbivert.com/on-the-theoretical-minimum-solutions/ or github.com/mbivert/ttm

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Exercise 1. *Show that the right-hand side of Eq. 9.17 simplifies to the right-hand side of Eq. 9.16. Hint: First expand the second term by taking the derivative of the product. Then look for cancellations.*

Eq. 9.17 and Eq. 9.16 are respectively:

$$[\boldsymbol{V}(x), \boldsymbol{P}]\psi(x) = V(x)(-i\hbar\frac{d}{dx})\psi(x) - (-i\hbar\frac{d}{dx})V(x)\psi(x)$$

$$[\boldsymbol{V}(x), \boldsymbol{P}] = i\hbar\frac{dV(x)}{dx}$$

Let's start as suggested by expanding the second term of Eq. 9.17, ignoring the $-i\hbar$ factor for now: this is a basic product rule application:

$$(\frac{d}{dx})(V(x)\psi(x)) = \frac{dV(x)}{dx}\psi(x) + V(x)\frac{d\psi(x)}{dx}$$

The second term will cancel with the first term of the RHS of Eq. 9.17:

$$\begin{aligned}
[\boldsymbol{V}(x), \boldsymbol{P}]\psi(x) = &\quad V(x)(-i\hbar\frac{d}{dx})\psi(x) - (-i\hbar\frac{d}{dx})V(x)\psi(x) \\
= &\quad -i\hbar V(x)\frac{d\psi(x)}{dx} + i\hbar\left(\frac{dV(x)}{dx}\psi(x) + V(x)\frac{d\psi(x)}{dx}\right) \\
= &\quad i\hbar\frac{dV(x)}{dx}\psi(x)
\end{aligned}$$

As long as $\psi(x) \neq 0$, we can divide by $\psi(x)$, and indeed establish Eq. 9.16:

$$\boxed{[\boldsymbol{V}(x), \boldsymbol{P}] = i\hbar\frac{dV(x)}{dx}} \quad \square$$